Divisibility of the Sum of Three Consecutive Integers

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Definition 1 (Divisibility)**.** *We say $a \mid b$ ("a* divides *b") if there exists some $k \in \mathbb{Z}$ such that*

$$b = k \cdot a.$$

Theorem 1. *The sum of any three consecutive integers is divisible by* 3*.*

Proof. Let $n \in \mathbb{Z}$, and let $n,\ n+1,\ n+2$ be three consecutive integers. Their sum is

$$n + (n+1) + (n+2) = 3n + 3 = 3(n+1).$$

Since $n + 1 \in \mathbb{Z}$, taking $k = n + 1$ in Definition 1 yields $3 \mid 3(n+1)$, and hence the sum is divisible by 3. □