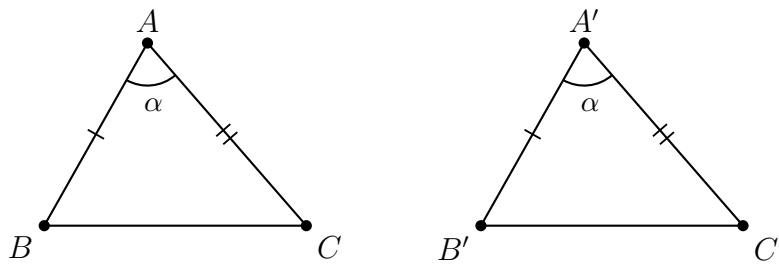


Congruence Criteria for Triangles

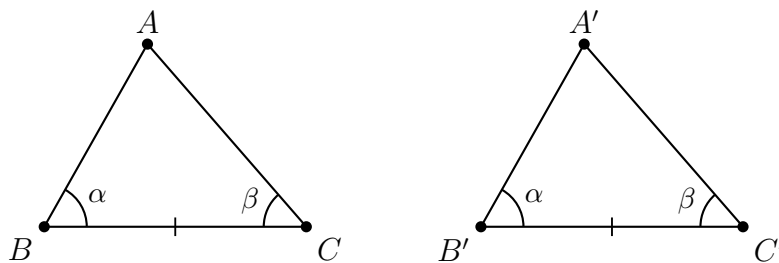
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April 11, 2026

First Criterion of Congruence (SAS). Two triangles are congruent if they have two sides and the included angle between them respectively congruent.



Second Criterion of Congruence (ASA). Two triangles are congruent if they have one side and the two angles adjacent to it respectively congruent.

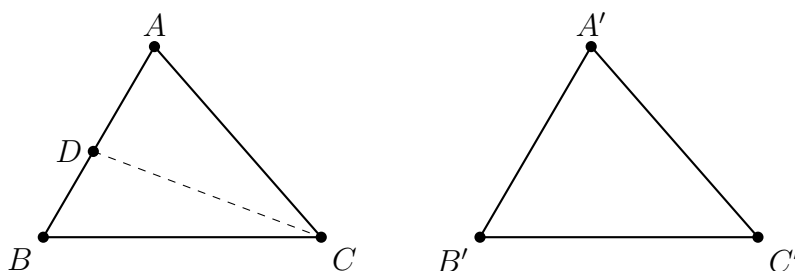


Proof of the Second Criterion

Let ABC and $A'B'C'$ be two triangles such that

$$AC \cong A'C', \quad \hat{ACB} \cong \hat{A'C'B'}, \quad \hat{CAB} \cong \hat{C'A'B'}.$$

We want to show that $\triangle ABC \cong \triangle A'B'C'$.



Suppose, for contradiction, that the two triangles are *not* congruent. Since $AC \cong A'C'$ by hypothesis, the non-congruence must involve at least one of the remaining sides; assume without loss of generality that $AB > A'B'$. In this case we can take a point D on segment AB such that $AD \cong A'B'$.

Consider now triangles ADC and $A'B'C'$. They satisfy:

1. $AD \cong A'B'$, by construction;
2. $AC \cong A'C'$, by hypothesis;
3. $\widehat{CAD} \cong \widehat{C'A'B'}$, by hypothesis (since D lies on ray AB , the angle \widehat{CAD} coincides with \widehat{CAB}).

By the **First Criterion of Congruence** (SAS), it follows that $\triangle ADC \cong \triangle A'B'C'$.

In particular, this congruence gives $\widehat{ACD} \cong \widehat{A'C'B'}$. But by hypothesis $\widehat{A'C'B'} \cong \widehat{ACB}$, so

$$\widehat{ACD} \cong \widehat{ACB}.$$

This is a contradiction: since D lies *strictly* between A and B , the ray CD falls strictly inside the angle \widehat{ACB} , which forces $\widehat{ACD} < \widehat{ACB}$.

The hypothesis of non-congruence is therefore absurd, and we conclude that the two triangles are congruent. \square